

PhD course Mathematical Methods in Meteorology and Oceanography  
Spring semester 2021

Teachers: Erland Källén (EK) and Inga Koszalka (IK)

Literature:

Sadri Hassani, 2009: *Mathematical Methods For Students of Physics and Related Fields*. Springer Verlag, pp 831. (H2009)

Sadri Hassani, 2013: *Mathematical Physics, A Modern Introduction to Its Foundations (Second Edition)*. Springer Verlag, pp 1205. (H2013)

Robert Hilborn, 2000, *Chaos and Nonlinear Systems. an introduction for scientists and engineers (Second edition)*, p. 650 (Ebook SU library: <https://libris.kb.se/bib/11767309>) (H2000)

All books are available in electronic format via SUB. Only certain chapters are included, see courseplan below.

Examination: The examination will be in the form of selected problems from the course books and student presentations of various applications of nonlinear dynamics.

Lecture plan:

Wednesday 31 March 15-17 (IK, EK)

Introduction to course. Eigenvalues, eigenvectors, orthogonal polynomials, Fourier series. Ch 7.4, 7.5, 10.6 (H2009).

Wednesday 7 April 15-17 (EK)

Partial Differential Equations, classification, separation of variables, cylindrical and spherical coordinates. Ch 22.1, 22.2, 22.3, 22.4 (H2009), ch 21 (H2013).

Tuesday 13 April 15-17 (EK)

Second order linear differential equations, superposition, Wronskian. Ch 24.1, 24.2 (H2009).

Thursday 15 April 15-17 (EK)

Sturm-Liouville theory, adjoint operators, ch 24.5, 24.6 (H2009).

Tuesday 20 April 15-17 (EK)

Laplace's equation, cartesian coordinates, ch 25 (H2009).

Thursday 22 April 15-17 (EK)

Laplace's equation, spherical coordinates, Legendre polynomials. Ch 26 (H2009), ch 13.4 (H2013).

Tuesday 27 April 15-17 (EK)

Laplace's equation, cylindrical coordinates, Bessel functions. Ch 27 (H2009).

Thursday 29 April 15-17 (EK)

Heat equation. Ch 28.1 (H2009).

Tuesday 4 May 15-17 (EK)

Wave equation. Ch 28.3.

Thursday 6 May 15-17 (EK)  
Fourier transform. Ch 29.1 (H2009).

Tuesday 11 May 15-17 (EK)  
Green's functions. Ch 29.2 (H2009).

Tuesday 18 May 15-17 (IK)  
Nonlinear dynamics and onset of chaos  
Ch 31.1-31.2 (H2009)  
Ch II (H2000)  
(Nonlinear systems. Differential equations and Iterated maps. Stable and Unstable Fixed Points.  
Phase Space, Bifurcation, Onset of chaos)

Thursday 20 May 15-17 (IK)  
Chaotic systems Ch 31.2-31.3 (H2009)  
Ch III (H2000)  
(Examples chaotic systems, Phase space, Lyapunov exponents & dimension, stretching and folding,  
entropy, Information)

Tuesday 25 May 15-17 (IK)  
The Lorenz model and its legacy  
Appendix C (H2000)  
Lorenz, E. N. (1963). Deterministic Nonperiodic Flow, *Journal of Atmospheric Sciences*, 20(2), 130-141. Retrieved Mar 11, 2021, from [https://journals.ametsoc.org/view/journals/atmsoc/20/2/1520-0469\\_1963\\_020\\_0130\\_dnf\\_2\\_0\\_co\\_2.xml](https://journals.ametsoc.org/view/journals/atmsoc/20/2/1520-0469_1963_020_0130_dnf_2_0_co_2.xml)

June 15, 15-17 (IK, EK)  
Student presentations: selected topics on applications of chaos theory, e.g.:  
Kapitza pendulum  
Pattern formation in biological systems  
Chaotic dynamics of ENSO  
Chaotic measures for oceanic turbulence  
Predictability in atmosphere  
Glacial cycles